

Nonperturbative screening of the Landau pole

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Abstract

Based on the trace anomaly for the energy-momentum tensor, an effective theory for the thermodynamics of the deconfining phase, and by assuming the asymptotic behavior to be determined by one-loop perturbation theory we compute the nonperturbative beta function for the fundamental coupling g in SU(2) and SU(3) Yang-Mills theory. With increasing temperature we observe a very rapid approach to the perturbative running. The Landau pole is nonperturbatively screened.

1 Introduction

Knowledge on the resolution dependence (running) of the fundamental coupling in Yang-Mills theories was first obtained within a perturbative setting [1]. This is important because the perturbative renormalizability of these theories [2], see also [3], states that (apart from a wave-function renormalization) the coupling remains the only parameter of the theory in the process of successive perturbative elimination of quantum fluctuations. The proof of the perturbative renormalizability of Yang-Mills theories belongs to the deepest and most far-reaching theoretical results of the last century. As a consequence, the notion of asymptotic freedom was established for the strong interactions governed by the gauge group $SU(3)$ [1]. The asymptotic freedom of Quantum Chromodynamics allows for a controlled small-coupling expansion of correlation functions around the conformal limit which takes place at an infinitely large resolution.

Nonperturbative investigations on the running of the gauge coupling have been carried out in the framework of the exact renormalization group equation (ERGE) and the approach via the Dyson-Schwinger equations, see [4, 5] and references therein. As it seems, these results indicate that the perturbatively derived Landau pole is regularized by nonperturbative effects. The purpose of the present article is to re-investigate this issue in an independent way.

On the one hand, we use a nonperturbative approach to $SU(2)$ and $SU(3)$ Yang-Mills thermodynamics [6], which predicts for the deconfining phase the existence of a unique, maximal resolution in terms of the modulus $|\phi|(T)$ of an emergent, adjoint Higgs field¹. On the other hand, we combine it with a nonperturbative definition of the gauge-coupling evolution via the trace anomaly for the energy-momentum tensor at finite temperature. This allows for an extraction of the nonperturbative rate of change of the fundamental coupling g under a variation of temperature: The beta-function.

As a boundary condition we require that the high-temperature behavior of the beta function is in accord with the perturbative situation. The so-extracted law governing the running of the coupling is in agreement with that obtained in one-loop perturbation theory (when setting the renormalization scale equal to temperature) except closely above the critical temperature.

The article is organized as follows. In Sec. 2 we briefly review the trace anomaly for the energy-momentum tensor and discuss its validity at finite temperature. Based on the trace anomaly and an effective theory for the thermodynamics of the deconfining phase the derivation of the nonperturbative evolution equation for the fundamental coupling in dependence of temperature is performed for $SU(2)$ and $SU(3)$ pure Yang-Mills theories in Sec. 3. We discuss the high-temperature limit to make contact with perturbation theory and subsequently solve the evolution equations. In Sec. 4 we summarize and discuss our results.

¹The spatially infinitely extended thermalized Yang-Mills system can be considered as an extended vertex which selfconsistently picks its own scale of maximal resolution [6].

2 Trace anomaly

The trace anomaly for the energy-momentum tensor $\theta_{\mu\nu}$, which is considered an operator identity, reads [7, 8]

$$\theta_{\mu\mu} = \frac{\beta(g)}{2g} F_{\mu\nu}^a F_{\mu\nu}^a, \quad (1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c$ is the field strength appearing in the *fundamental* Lagrangian of the Yang-Mills theory, $\mathcal{L}_{YM} = -\frac{1}{4}(F_{\mu\nu}^a)^2$, the f^{abc} are the structure constants of the Lie algebra, and β is given by the right-hand side of the evolution equation for the *fundamental* gauge coupling g :

$$\mu \partial_\mu g = \beta(g). \quad (2)$$

In Eq. (2) the mass scale μ refers to the resolution that is applied to the process at which the strength of the coupling g is extracted. In contrast to the chiral anomaly [7], which is not renormalized because of its topological nature, the trace anomaly exhibits two resolution-dependent factors: The β -function divided by g and the average of $F_{\mu\nu}^a F_{\mu\nu}^a$.

When solving $\beta(g) = \mu \partial_\mu g = -bg^3$ ($b = \frac{11N}{48\pi^2}$, valid at one-loop order and zero-temperature) a Landau pole $\mu = \Lambda_L$, which roughly is identified with the Yang-Mills scale Λ ($\Lambda_L \sim \Lambda$), occurs:

$$\Lambda_L = \mu_0 \exp\left(-\frac{1}{2bg_0^2}\right). \quad (3)$$

Eq. (3) then implies the well-known form for the running coupling:

$$g^2(\mu) = \frac{g_0^2}{1 + 2b \ln(\frac{\mu}{\mu_0})} = \frac{1}{2b \ln(\frac{\mu}{\Lambda_L})}. \quad (4)$$

Let us now turn to the case of finite temperature: We work in a flat Euclidean spacetime with time τ constrained as $0 \leq \tau \leq 1/T$. In [9] the one-loop zero-temperature expression in Eq. (3) was used to argue for the validity² of Eq. (1) when taking a *thermal* average.

Performing a thermal average over Eq. (1), we obtain

$$\rho - 3p = \frac{\beta(g)}{2g} \langle F_{\mu\nu}^a F^{a,\mu\nu} \rangle_T, \quad (5)$$

²The grand canonical potential density, $\frac{\Omega}{V} = -P$, must be expressible as $\Lambda^4 f(\Lambda/T)$ where f is a dimensionless function of its dimensionless argument. Taking the derivative of $\frac{\Omega}{V}$ with respect to the exponent in Eq. (3), the thermal average over both sides of Eq. (1) appears with the one-loop level expression for β . The authors of [9] then claim that there is no difficulty to extend this result to an arbitrary order in perturbation theory. Here we will consider the one-loop situation only.

where ρ and p describe the energy density and the pressure of the thermalized Yang-Mills system. Notice that in Eq. (5) two scales enter: the temperature T , at which the thermal averages are calculated, and the scale μ , which is the resolution at which $\beta(g) = \mu \partial_\mu g$ is evaluated [10, 11]. In our approach [6, 12, 13] the scales μ and T are not independent but functionally related.

3 Nonperturbative β -function

3.1 Effective SU(2) Yang-Mills thermodynamics

We now turn to the effective theory for SU(2)-thermodynamics in the deconfining phase. There are topologically trivial, coarse-grained gauge fields a_μ entering in the effective field strength $G_{\mu\nu}^a = \partial_\mu(a_\nu^a) - \partial_\nu(a_\mu^a) - e\epsilon^{abc}a_\mu^b a_\nu^c$ (to be distinguished from the fundamental field strength $F_{\mu\nu}^a$ in $\mathcal{L}_{YM} = -\frac{1}{4}(F_{\mu\nu}^a)^2$), and there is an inert, adjoint scalar field ϕ , which together with a pure-gauge configuration represents the thermal ground state emerging from a spatial average over interacting calorons and anticalorons of topological charge modulus $|Q| = 1$. The effective coupling e is temperature dependent (to be distinguished from the fundamental coupling g). The dependence $e = e(T)$ follows from thermodynamical selfconsistency, see below. The effective Lagrangian for the description of SU(2)-Yang-Mills thermodynamics in the deconfining phase ($T > T_c = \lambda_c \Lambda / 2\pi$, $\lambda_c = 13.87$) and in unitary gauge reads [6, 13]:

$$\mathcal{L}_{\text{dec-eff}}^{u.g.} = \frac{1}{4} (G_E^{a,\mu\nu}[a_\mu])^2 + 2e(T)^2 |\phi|^2 \left((a_\mu^{(1)})^2 + (a_\mu^{(2)})^2 \right) + \frac{2\Lambda_E^6}{|\phi|^2}. \quad (6)$$

The modulus of the adjoint scalar field $|\phi|$ depends on the Yang-Mills scale Λ and on temperature T as $|\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$. The length $|\phi|^{-1}$ is the minimal length down to which the thermalized system looks spatially homogeneous. In other words, the spatial average over interacting calorons and anticalorons selfconsistently saturates at this length scale. The quantum fluctuations $a_\mu^{(1,2)}$ are massive in a temperature dependent way, $m^2 = m(T)^2 = 4e^2 |\phi|^2$, while the gauge mode $a_\mu^{(3)}$, stays massless (dynamical gauge symmetry breaking: $SU(2) \rightarrow U(1)$).

We work with the following dimensionless quantities

$$\bar{\rho} = \frac{\rho}{T^4}, \quad \bar{p} = \frac{p}{T^4}, \quad \lambda = \frac{2\pi T}{\Lambda}, \quad a(\lambda) = \frac{m(T)}{T} = 2 \frac{e(T)}{T} |\phi| \quad (7)$$

where ρ and p are the energy density and the pressure due to the Lagrangian (6), and the function $a = a(\lambda)$ is introduced for later use.

The energy density and pressure ρ and p are a sum of three contributions

$$\rho = \rho_3 + \rho_{1,2} + \rho_{gs}, \quad p = p_3 + p_{1,2} + p_{gs}, \quad (8)$$

where the subscript 1,2 is understood as a sum over the two massive modes $a_\mu^{(1,2)}$, the subscript 3 refers to the massless mode $a_\mu^{(3)}$, and the subscript gs labels the ground-state contribution. When expressing $\bar{\rho}$ and \bar{p} as functions of the dimensionless temperature λ , one obtains at one-loop (accurate on the 0.1%-level [6, 14]):

$$\bar{\rho}_3 = 2\frac{\pi^2}{30}, \quad \bar{\rho}_{1,2} = \frac{3}{\pi^2} \int_0^\infty dx \frac{x^2 \sqrt{x^2 + a^2}}{e^{\sqrt{x^2 + a^2}} - 1}, \quad \bar{\rho}_{gs} = \frac{2(2\pi)^4}{\lambda^3}. \quad (9)$$

$$\bar{p}_3 = 2\frac{\pi^2}{90}, \quad \bar{p}_{1,2} = -\frac{3}{\pi^2} \int_0^\infty x^2 dx \ln \left(1 - e^{-\sqrt{x^2 + a^2}} \right), \quad \bar{p}_{gs} = -\bar{\rho}_{gs}. \quad (10)$$

Imposing the validity of the thermodynamical Legendre transformation

$$\rho = T \frac{dP}{dT} - P \iff \bar{\rho} = \lambda \frac{d\bar{p}}{d\lambda} + 3\bar{p} \quad (11)$$

and substituting the expressions (9)-(10) into (11), we arrive at the following differential equation for $a = a(\lambda)$:

$$1 = -\frac{6\lambda^3}{(2\pi)^6} \left(\lambda \frac{da}{d\lambda} + a \right) a D(a), \quad (12)$$

$$D(a) = \int_0^\infty dx \frac{x^2}{\sqrt{x^2 + a^2}} \frac{1}{e^{\sqrt{x^2 + a^2}} - 1}, \quad a(\lambda_{in}) = 0. \quad (13)$$

For a sufficiently large initial value λ_{in} the solution for $a(\lambda)$ is independent on λ_{in} : a low-temperature attractor with a logarithmic pole at $\lambda_c = 13.87$ is seen to exist. The effective coupling is given as $e = e(\lambda) = a(\lambda)\lambda^{3/2}/4\pi$, and exhibits a plateau $e = \sqrt{8}\pi$ for $\lambda \gg \lambda_c$. In fact,

$$a(\lambda) = \frac{8\sqrt{2}\pi^2}{\lambda^{3/2}} \quad (14)$$

is a solution of the differential equation (12) for $a \ll 1$, that is, for $\lambda \gg \lambda_c$. For plots and the discussion of the thermodynamical quantities we refer to [6], and for the discussion of the linear growth of $\langle \theta_{\mu\mu} \rangle_T$ to [15].

3.2 Nonperturbative running coupling $g(T)$: $SU(2)$ -case

We now use the effective Lagrangian (6) to evaluate the two relevant elements of the trace-anomaly equation

$$\rho - 3p = \frac{\beta(g)}{2g} \langle F_{\mu\nu}^a F^{a,\mu\nu} \rangle_T. \quad (15)$$

Namely,

- (i) $\rho - 3p$ is evaluated from Eqs. (9)-(10).
- (ii) The expectation value $\langle F_{\mu\nu}^a F^{a,\mu\nu} \rangle_T$ is the average action density in euclidean space (a possible definition of the gluon condensate as discussed in [13, 16]).

We evaluate the average action density by utilizing the effective Lagrangian (6) considering that the part of the fundamental field strength $F_{\mu\nu}^a$, which enters the ground-state physics described by the effective theory³, suffers a wave-function renormalization. We have

$$\langle \mathcal{L}_{\text{YM}} \rangle_T = \frac{1}{4} \langle F_{\mu\nu}^a F^{a,\mu\nu} \rangle_T = f^2(g) \langle \mathcal{L}_{\text{dec-eff}}^{u.g.} \rangle_T = f^2(g) \rho_{gs} \quad (16)$$

with $\rho_{gs} = \lambda^4 \overline{\rho}_{gs} = 4\pi\Lambda^3 T$. The function $f(g)$ will be fixed by requiring that for $\lambda \gg \lambda_c$ the fundamental coupling g runs in agreement with perturbation theory.

Within the effective theory we can relate the natural (not externally imposed) resolution scale μ to temperature T . We have $\mu = |\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$ [6, 12, 13]. That is, the thermalized Yang-Mills system acts like a spatially extended vertex being probed with a selfconsistently adjusting resolution $|\phi|$ as soon as a temperature T is provided. Fluctuations, that would be resolved at $\mu > |\phi|$, are integrated out in the effective theory. Thus we have:

$$\beta(g) = \mu \partial_\mu g = -2T \partial_T g = -2\beta_T(g), \quad (17)$$

where $\beta_T(g) \equiv T \partial_T g$. Notice that $\beta(g) = \mu \partial_\mu g$ is a positive quantity: In fact, the trace anomaly $\rho - 3p$ and $\langle F_{\mu\nu}^a F^{a,\mu\nu} \rangle_T$ are both positive. The fact that the resolution $\mu = |\phi|$ decreases for increasing T generates a negative $\beta_T(g)$ in accord with asymptotic freedom ($T \gg T_c$).

Taking into Eqs. (15), (16), and (17), we have

$$h(\lambda) \equiv \frac{\rho - 3p}{4\rho_{gs}} = -\frac{\beta_T(g)}{g} f^2(g). \quad (18)$$

The function $h(\lambda)$, see also [15], is plotted in Fig. 1. Notice that h approaches the value $\frac{3}{2}$ for $\lambda \gg \lambda_c$ thus implying that $\langle \theta_{\mu\mu} \rangle_T = \rho - 3p = 6\rho_{gs} = 24\pi\Lambda^3 T$ for $T \gg T_c$. This simple high- T behavior allows to determine the function $f(g)$ analytically. We require that the perturbative result $\beta_T(g) = -bg^3$ ($b = \frac{11N}{48\pi^2}$, $N = 2$) holds for $g \ll 1$ (or $T \gg T_c$). Then Eq. (18) implies that

$$f(g) = \sqrt{\frac{3}{2b}} \frac{1}{g}. \quad (19)$$

The evolution equation (18) for $g = g(\lambda)$ is recast as:

$$\beta_T(g) = -\frac{2}{3} b h(\lambda) g^3 \Leftrightarrow \partial_\lambda g = -\frac{2}{3} b \frac{h(\lambda)}{\lambda} g^3. \quad (20)$$

³On the one hand, it is straight-forward to show that on-shell excitations do not contribute to the thermal average $\langle \mathcal{L}_{\text{dec-eff}} \rangle_T$ in Eq. (6) on the one-loop level. On the other hand, in the effective theory quantum fluctuations make a negligible contribution as compared to that of the ground state [6].

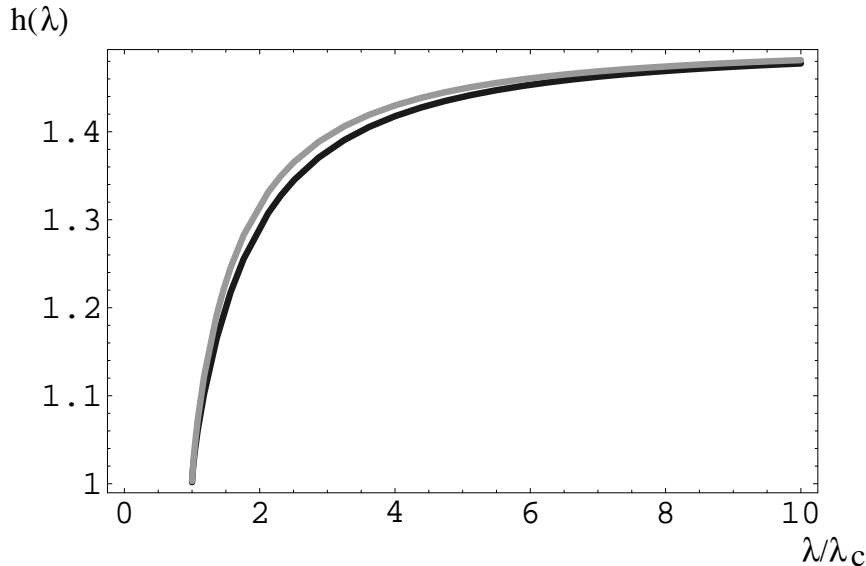


Figure 1: The function $h(\lambda)$, defined in Eq. (18), plotted for the SU(2) (gray curve) and for the SU(3) (black curve) Yang-Mills theories.

From the behavior of $h(\lambda)$ we can immediately infer two interesting properties:

- a) The function $h(\lambda) \simeq \frac{3}{2}$ for $\lambda > 5\lambda_c$. That is, the perturbative equation $\beta_T(g) = -bg^3$ is valid all the way down to $5\lambda_c$. The range of validity of the perturbative treatment for the determination of $g(\lambda)$ is thus even larger than one naively would think. Indeed, as we will see later, the perturbative solution is very similar to the nonperturbative one even down to temperatures $\sim 1.2T_c$.
- b) The function $h(\lambda)$ slowly decreases for decreasing temperatures thus effectively lowering the coefficient b in the perturbative beta function. Therefore a mild screening of the perturbative Landau pole occurs.

To solve Eq. (20) an initial condition must be specified. As discussed in the previous subsection, the effective coupling constant $e(\lambda)$ shows a logarithmic pole at the critical temperature $\lambda_c = 13.87$. We therefore impose that the fundamental coupling $g(\lambda)$ also diverges at the deconfining temperature. In this way the solution $g = g(\lambda)$ is uniquely fixed. Fig. 2 shows this solution and the perturbative solution $g_P(\lambda)$ of Eq. (4) (by setting $\mu = \lambda$). The boundary condition for $g_P(\lambda)$ is determined by imposing that g and g_P coincide at large values of λ . In practice, we can set $g(\lambda = 10\lambda_c) = g_P(\lambda = 10\lambda_c)$ (another choice of the matching at high T clearly would lead to very similar results). Notice that in Fig. 2 the perturbative Landau pole is to the right of λ_c . Also, the nonperturbative coupling g is screened as compared to g_P : $g(\lambda)$ is always below $g_P(\lambda)$. Before moving on to the SU(3) case one comment is in order. Namely, our determination of the wave-function renormalization $f(g)$ is based on the one-loop leading, asymptotic behavior of the beta function. Higher-order perturbative corrections, however, depend on the adopted renormalization scheme. Demanding scheme-invariance of the nonperturbative coupling, we are left with the one-loop expression for $f(g)$.

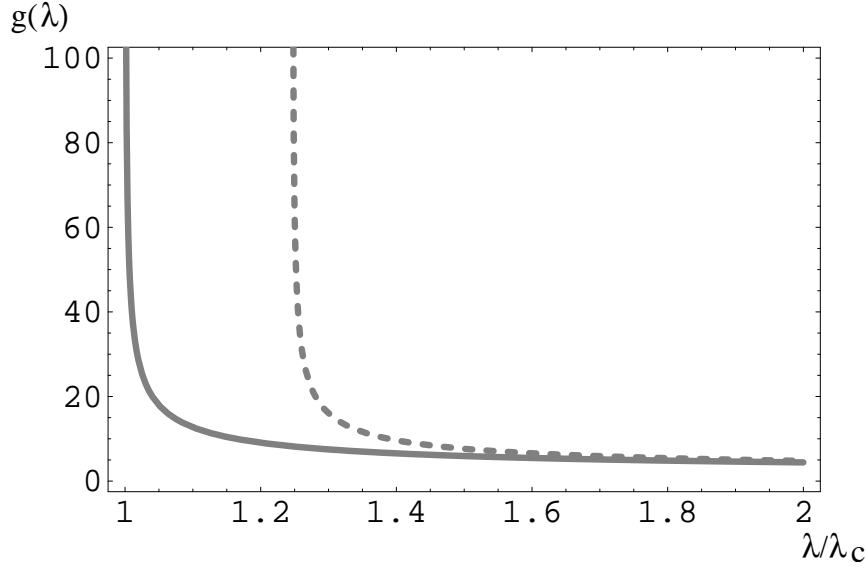


Figure 2: The running of the nonperturbative $g(\lambda)$ (undotted) and that of the perturbative counterpart $g_P(\lambda)$ (dotted) for the SU(2) case.

3.3 Nonperturbative running coupling $g(T)$: $SU(3)$ -case

Here the effective thermodynamic description follows the same lines as in the SU(2) case, see [6]. We only report on some relevant formulas and briefly discuss their consequences. The modulus of the scalar field ϕ is exactly the same. As shown in [6] out of the eight coarse-grained gauge modes four acquire a mass $m_1 = e|\phi|$ (contributing to ρ and p by ρ_1 and p_1), two a mass $m_2 = 2e|\phi|$ (ρ_2 and p_2), and two stay massless (ρ_3 and p_3). Explicitly, we have ($a = m_1/T$):

$$\bar{\rho}_3 = 4 \frac{\pi^2}{30}, \quad \bar{\rho}_1 = \frac{6}{\pi^2} \int_0^\infty dx \frac{x^2 \sqrt{x^2 + a^2}}{e^{\sqrt{x^2 + a^2}} - 1}, \quad (21)$$

$$\bar{\rho}_2 = \frac{3}{\pi^2} \int_0^\infty dx \frac{x^2 \sqrt{x^2 + (2a)^2}}{e^{\sqrt{x^2 + (2a)^2}} - 1}, \quad \bar{\rho}_{gs} = \frac{2(2\pi)^4}{\lambda^3}, \quad (22)$$

$$\bar{p}_3 = 4 \frac{\pi^2}{90}, \quad \bar{p}_1 = -\frac{6}{\pi^2} \int_0^\infty dx x^2 \ln \left(1 - e^{-\sqrt{x^2 + a^2}} \right), \quad (23)$$

$$\bar{p}_2 = -\frac{3}{\pi^2} \int_0^\infty dx x^2 \ln \left(1 - e^{-\sqrt{x^2 + (2a)^2}} \right), \quad \bar{p}_{gs} = -\bar{\rho}_{gs}. \quad (24)$$

Thermodynamical selfconsistency, Eq. (11), implies that

$$1 = -\frac{12\lambda^3}{(2\pi)^6} \left(\lambda \frac{da}{d\lambda} + a \right) (aD(a) + 2aD(2a)). \quad (25)$$

The asymptotic solution to Eq. (25) reads $a(\lambda) = \frac{8}{\sqrt{3}}\pi^2\lambda^{-3/2}$, and the effective coupling reaches a plateau value of $e = \frac{4}{\sqrt{3}}\pi$ for $\lambda \gg \lambda_c$. The value for λ_c , where the effective coupling e possesses a logarithmic pole, is $\lambda_c = 9.475$ [6]. The asymptotic value of the function $h(\lambda)$, defined in (18), is $3/2$ just like in the SU(2) case. The function $h(\lambda)$ is plotted in Fig. 1, and an analogous behavior to the SU(2) curve is evident.

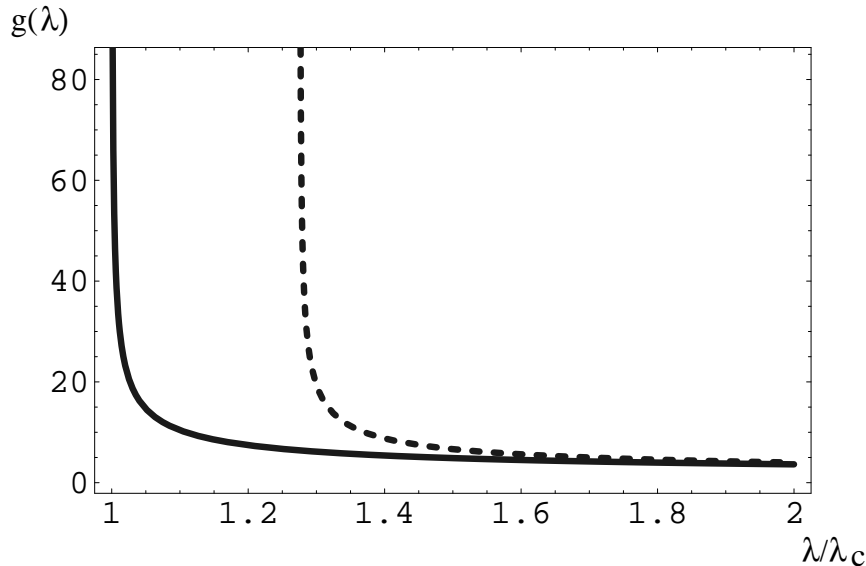


Figure 3: The running of the nonperturbative coupling $g(\lambda)$ (undotted) and the perturbative counterpart $g_P(\lambda)$ (dotted) in the SU(3) case.

Equations (19) and (20) hold for $b = \frac{11N}{48\pi^2}$, $N = 3$. The qualitative discussion is very similar to the SU(2) case. The nonperturbative and the perturbative solutions are plotted in Fig. 3 where a screening of the perturbative Landau pole is observed. The similarity of the SU(2) and SU(3) cases is remarkable.

4 Summary and Conclusions

We have computed the beta-function for the fundamental coupling in SU(2) and SU(3) Yang-Mills theory by appealing to the trace anomaly for the energy-momentum tensor and an effective theory for the thermodynamics of the deconfining phase. The latter involves a adjoint scalar field ϕ which emerges upon a coarse-graining process performed over interacting calorons and anticalorons [6] and together with a pure-gauge configuration represent the thermal ground state for the system. In contrast to earlier (perturbative) investigations, where a separate dependence of the coupling on the resolution μ and temperature T was employed [10, 11], the effective theory dictates the relevant μ at a given T in terms of the modulus of the scalar field $|\phi|$. Therefore T and μ are no longer independent variables, but they are functionally related: $\mu = |\phi| = \sqrt{\frac{\Lambda^3}{2\pi T}}$ where Λ is the Yang-Mills scale. The beta-function, defined by the rate of change of the fundamental coupling g when varying the resolution scale μ , thus also relates to the rate of change β_T of g when varying T . This enables a comparison with the one-loop, zero-temperature perturbative prediction (obtained by setting $\mu = T$). In doing so we have assumed that for asymptotically large temperatures the full beta function exhibits the universal, perturbative one-loop behavior.

Remarkably, already for temperatures slightly greater than the critical temperature, we observe the behavior of β_T as predicted by one-loop perturbation theory [1]. Deviations become apparent only for smaller temperature, say below $1.5 T_c$. The nonperturbative effects generate a screening of the perturbative Landau pole: The

nonperturbative pole is slightly shifted to the left of the perturbative Landau pole for both SU(2) and SU(3).

The fact that in our study the perturbative solution is valid up to temperatures slightly above T_c relies on the properties of the function $h(\lambda) = \frac{\rho-3p}{4\rho_{gs}}$ as plotted in Fig. 1 ($\lambda = \frac{2\pi}{T}$ and ρ_{gs} is the explicit contribution of the ground state to the energy density). At high T the asymptotic behavior is $h(\lambda) \simeq 3/2$, and the perturbative evolution equation is recovered. In the context of the present paper, the very presence of the nonperturbative ground state is ultimately responsible of this simple asymptotic behavior of $h(\lambda)$. Notice that the asymptotic behavior for $h(\lambda)$ implies that $\rho - 3p = 6\rho_{gs} = 24\pi\Lambda^3 T$. Thus the trace anomaly grows linearly with temperature for sufficiently high T , see also [15, 17, 18].

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